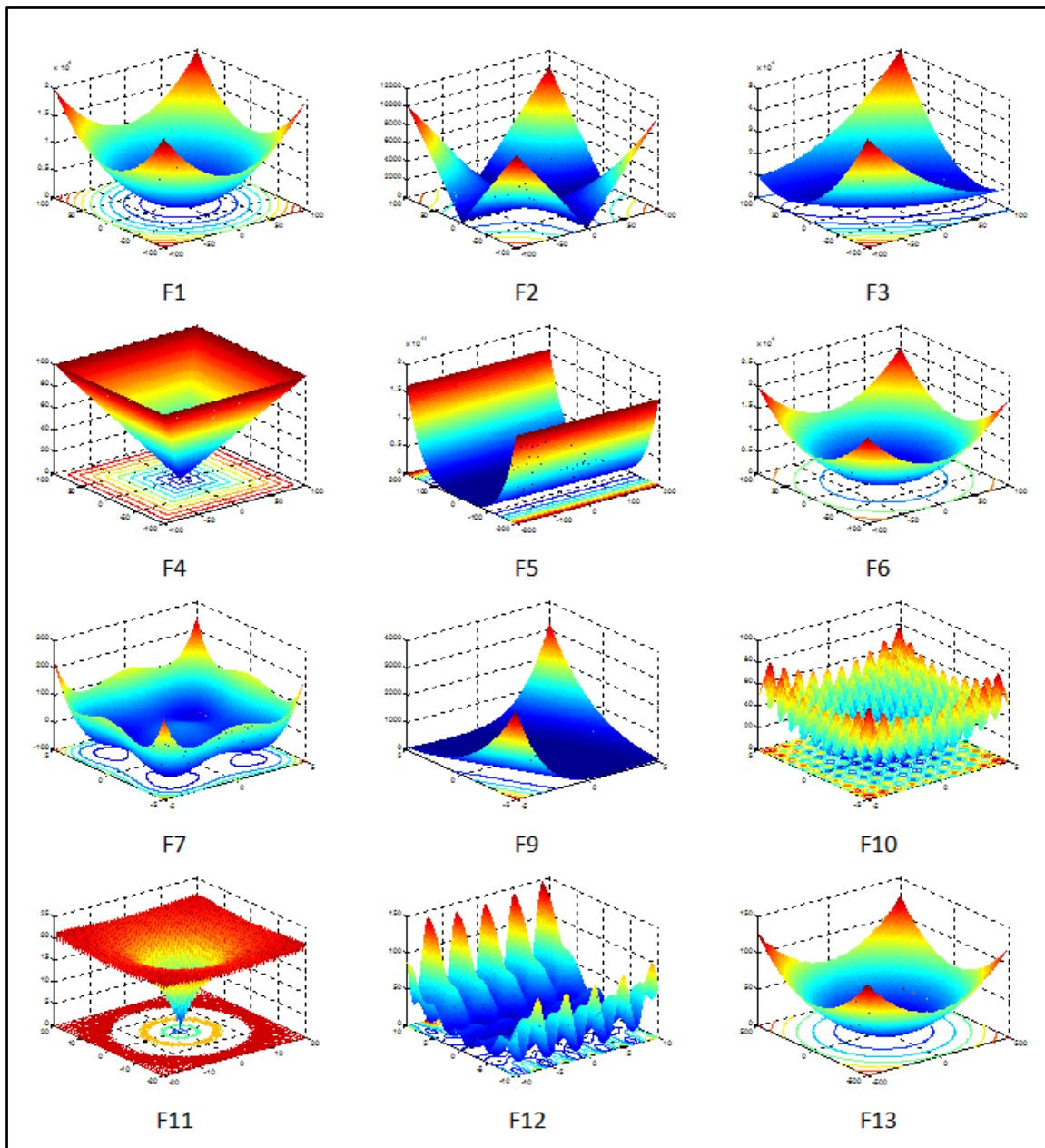# Benchmark Functions

In this Section we are presenting the benchmark functions that are used in this work.

Table 1 Unimodal and multimodal Benchmark functions
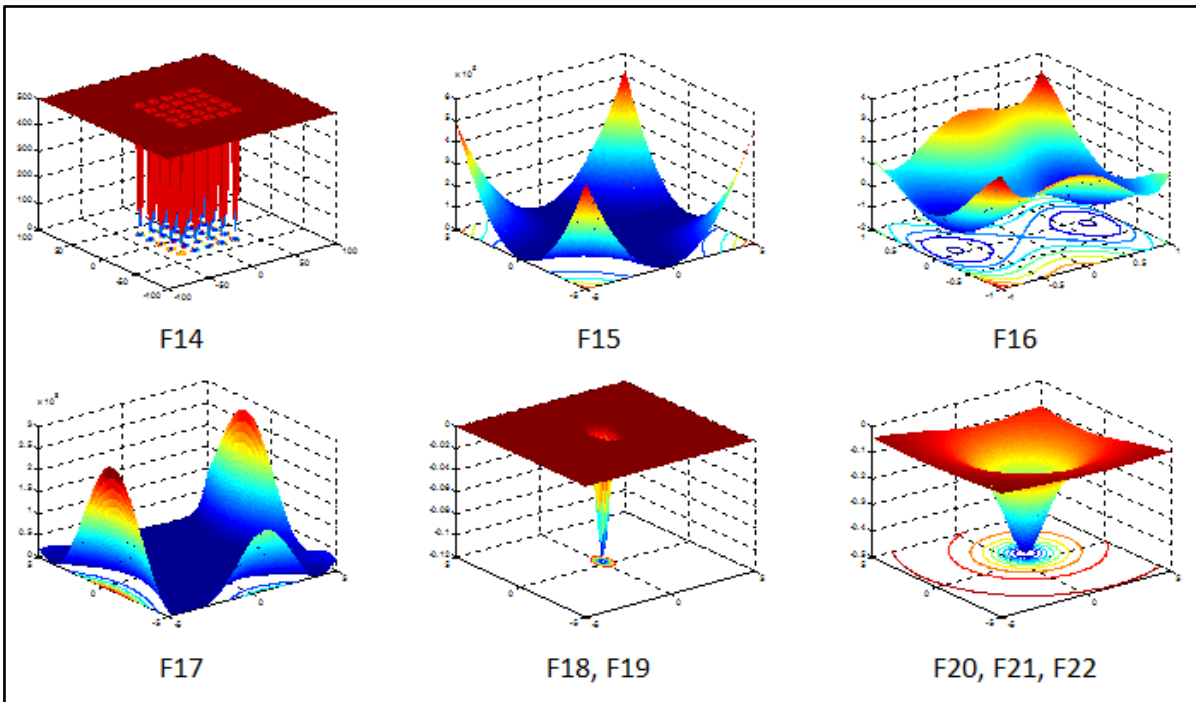
Function	Range	f_{min}
$f_1(x) = \sum_{i=1}^{n} x_i^2$	[-100, 100]	0
$f_2(x) = \sum_{i=1}^{n}\lvert x_1 \rvert + \prod_{i=1}^{n}\lvert x_1 \rvert$	[-100, 100]	0
$f_3(x) = \sum_{i=1}^{n}\left(\sum_{j-1}^{i} x_j\right)^2$	[-100, 100]	0
$f_4(x) = max_i\{\lvert x_1 \rvert, 1 \leq i \leq n\}$	[-100, 100]	0
$f_5(x) = \sum_{i=1}^{n-1}\left[100(x_{i+1} - x_i^2)^2 + (x_1 - 1)^2\right]$	[-100, 100]	0
$f_6(x) = \sum_{i=1}^{n}\left(\left[x_{-_1} + 0.5\right]\right)^2$	[-100, 100]	0
$f_7(x) = \frac{1}{2}\sum_{i=1}^{d}(x_i^4 - 16x_i^2 + 5x_i)$	[-100, 100]	-39.17
$f_8(x) = \sum_{i=1}^{d/4}\begin{bmatrix}(x_{4i-3} + 10x_{4i-2})^2 + 5(x_{4i-1} - x_{4i})^2 \\ + (x_{4i-2} - 2x_{4i-1})^4 + 10(x_{4i-3} - x_{4i})^4\end{bmatrix}$	[-100, 100]	0
$f_9(x) = \sum_{i=1}^{d} x_i^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^2 + \left(\sum_{i=1}^{d} 0.5ix_i\right)^4$	[-100, 100]	0
$f_{10}(x) = \sum_{i=1}^{n}[x_i^2 - 10\cos(2\pi x_i) + 10]$	[-100, 100]	0
$f_{11}(x) = 20\,exp\left(-0.2\sqrt{\frac{1}{n}\sum_{i=1}^{n} x_i^2}\right) - exp\left(\frac{1}{n}\sum_{i=1}^{n}\cos(2\pi x_i)\right) + 20 + e$	[-100, 100]	0
$f_{12}(x) = \frac{\pi}{n}\left\{10\sin(\pi y_1) + \sum_{i=1}^{n-1}(y_i - 1)^2\left[1 + 10\sin^2(\pi y_{y+1})\right] + (y_n - 1)^2\right\} + \sum_{i=1}^{n} u(x_i, 10, 100, 4)$ $y_1 = 1 + \frac{x_i+1}{4}$ $u(x_1, a, k, m) = \begin{cases} k(x_i - a)^m, & x_i > a \\ 0, & -a < x_i < a \\ k(-x_i - a)^m, & x_i < -a \end{cases}$	[-100, 100]	0
$f_{13}(x) = \frac{1}{4000}\sum_{i=1}^{n} x_1^2 - \prod_{i=1}^{n}\cos\left(\frac{x_i}{\sqrt{i}}\right) + 1$	[-100, 100]	0

Table 2 Fixed-dimension multimodal benchmark functions

Function	Dim	Range	f_{min}
$f_{14}(x) = \left(\frac{1}{500} + \sum_{j=1}^{25} \frac{1}{j + \sum_{i=2}^{2}(x_i - a_{ij})^6}\right)^{-1}$	2	[-65, 65]	1
$f_{15}(x) = \sum_{i=1}^{11}\left[a_i - \frac{x_1(b_i^2 + b_i x_2)}{b_i^2 + b_i x_3 + x_4}\right]^2$	4	[-5, 5]	0.00030
$f_{16}(x) = 4x_1^2 - 2.1x_1^4 + \frac{1}{3}x_1^6 + x_1 x_2 - 4x_2^2 + 4x_2^4$	2	[-5, 5]	-1.0316
$f_{17}(x) = [1 + (x_1 + x_2 + 1)^2 (19 - 14x_1 + 3x_1^2 - 14x_2 + 6x_1 x_2 + 3x_2^2)] \times [30 + (2x_1 - 3x_2)^2 \times (18 - 32x_1 + 12x_1^2 + 48x_2 - 36x_1 x_2 + 27x_2^2)]$	2	[-2, 2]	3
$f_{18}(x) = -\sum_{i=1}^{4} c_1 \exp\left(-\sum_{j=1}^{3} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	3	[1, 3]	-3.86
$f_{19}(x) = -\sum_{i=1}^{4} c_1 \exp\left(-\sum_{j=1}^{6} a_{ij}\left(x_j - p_{ij}\right)^2\right)$	6	[0, 1]	-3.32
$f_{20}(x) = -\sum_{i=1}^{5}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.1532
$f_{21}(x) = -\sum_{i=1}^{7}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.4028
$f_{22}(x) = -\sum_{i=1}^{10}\left[(X - a_i)(X - a_i)^T + c_i\right]^{-1}$	4	[0, 10]	-10.5363

Fig. 1 Examples of the unimodal and multimodal benchmark functions in their 3D versions

Fig. 2 Examples of the fixed-dimension multimodal benchmark functions in their 3D versions